The use of waste aggregate in building industry
Numerical analysis of fibre reinforced slabs under impact loads.

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Fact:
• Due to lack of natural sources of gravel in Northern Poland the production of concrete becomes less economical and effective.
Fact:

• Exploitation of gravel sources leads to significant negative changes in environment, landscape and quality of life for residents. Necessity of transportation for long distances means the pollution of air, damages of roads and railways.

Fact:

• Demolition of existing building and other objects produces a growing amount of waste materials, mainly rubble.
One of possible solutions is to use the rubble (concrete and ceramic) together with fine sand and fibres as a substitute for conventional concrete.

Needs and requirements: technology of production, procedures for testing, methodology of numerical analysis (FEM – material models).

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Aim of this study: numerical simulation of experimental impact tests performed on circular plates, in order to determine the adequate nonlinear material model for concrete with rubble aggregate and fibre reinforcement.

Circular slabs: diameter 1 m, thickness 0.1 m, supported in three segments. Ceramic rubble (bricks) sorted and prepared. Steel fibres: 0 – 0.5 – 1 % (volume).
Load: free fall of weight 40 kg or 200 kg from various heights (from 0.2 to 1.0 m).

Slabs prepared for testing (64 slabs – various percentage of fibre reinforcement, amount of rubble, etc).
Additional testing of material samples: compression, shearing, bending, etc.

Crack propagated through thickness of the plate
Pattern of cracks – bottom view

Damaged plate – bottom view
Numerical analysis

Preliminary performed in order to evaluate the impact energy necessary to produce damage in slabs. This enabled to build the experimental stand, design supports, calculate weights, etc.

In this moment the numerical simulations are carried out to obtain the results consistent with experimental data, in terms of damage development and distribution.

Computer code: Abaqus Explicit.

Material models: CDP (Concrete Damage Plasticity – default in Abaqus), VUMAT (proposed by authors*)

* K. Cichocki, M. Ruchwa, Robustness oriented analysis of structures under extreme loads, CMM 2011, Warsaw, Poland

MATERIAL MODEL FOR CONCRETE – BASIC FEATURES

- Different compressive and tensile characteristics;
- Initial linear-elastic behaviour;
- Hardening/softening in compression;
- Softening in tension;
- Damage;
- Rate-dependent behaviour;

Tension  
Compression

**Rate-Dependent Plastic-Damage Material Model for Concrete**

- Continuum Damage Mechanics
  Kachanov (1958)

- Helmholtz free energy potential
  Lubliner (1972), Mazars and Pijaudier-Cabot (1989)

- Two independent internal scalar damage variables: \( d^+, d^- \)
  (tension, compression)
  Lemaitre (1984)

- Effective stress concept
  Lemaitre and Chaboche (1978)

- Rate dependent behavior
  Simo and Ju (1987)

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Kachanov, L.M. (1986), Introduction to Continuum Damage Mechanics, Martinus Nijhoff Publishers
Lemaitre, J. (1996), A Course on Damage Mechanics, Springer

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**Effective Stress \( \mathbf{\overline{\sigma}} \), Damage Parameter \( d \)**

\[
\mathbf{\sigma} = \frac{\mathbf{F}}{S} \\
\mathbf{\overline{\sigma}} = \frac{\mathbf{F}}{S - S_d} \\
\mathbf{\overline{\sigma}} = \frac{\mathbf{\sigma}}{1 - d}; \quad d = \frac{S_d}{S} \\
0 \leq d \leq 1
\]

General 3D representation:

\[
\mathbf{\overline{\sigma}} = \mathbf{D}_o : (\mathbf{\varepsilon} - \mathbf{\varepsilon}^p)
\]
Helmholtz free energy

\[ \Psi(\varepsilon, \varepsilon^p, d^+, d^-) = (1 - d^+)\Psi_0^+ (\varepsilon, \varepsilon^p) + (1 - d^-)\Psi_0^- (\varepsilon, \varepsilon^p) \]

\[ \Psi_0^+ = \Psi_0^+ (\bar{\sigma}(\varepsilon, \varepsilon^p)) = \frac{1}{2} \bar{\sigma}^+ : D_0^{-1} : \bar{\sigma} \]

\[ \Psi_0^- = \Psi_0^- (\bar{\sigma}(\varepsilon, \varepsilon^p)) = \frac{1}{2} \bar{\sigma}^- : D_0^{-1} : \bar{\sigma} \]

\[ \bar{\sigma}^+ = \langle \bar{\sigma} \rangle = \sum_{i=1}^{3} \langle \sigma_i \rangle p_i \otimes p_i \]

\[ \bar{\sigma}^- = \langle \bar{\sigma} \rangle = \sum_{i=1}^{3} \langle \sigma_i \rangle p_i \otimes p_i \]

Characterization of damage

Equivalent tensile stress

\[ \bar{\tau}^+ = \sqrt{\bar{\sigma}^+ : D_0^{-1} : \bar{\sigma}^+} \]

Equivalent compressive stress

\[ \bar{\tau}^- = \sqrt{\bar{\sigma}^- : D_0^{-1} : \bar{\sigma}^-} \]

\[ \bar{\sigma}_{oct} = \frac{1}{3} tr(\bar{\sigma}^-); \quad \bar{\tau}_{oct} = \sqrt{\frac{2}{3} J_2}; \quad K = \sqrt{2 \frac{1 - R_0}{1 - 2R_0}}; \quad R_0 = \frac{f_{0d}}{f_{0d}} \]

Damage criteria:

\[ g^+(\bar{\tau}^+, \bar{\tau}^+) = \bar{\tau}^+ - \bar{\tau}^+ \leq 0 \]

\[ g^-(\bar{\tau}^-, \bar{\tau}^+) = \bar{\tau}^- - \bar{\tau}^- \leq 0 \]
Evolution of damage variables (Oliver, 1990)

**Tension:**

$$d^+ = \tilde{r}^+ \frac{\partial G^+(\tilde{r}^+)}{\partial \tilde{r}^+} = \dot{G}^+ \geq 0$$

$$d^+ = 1 - \frac{r_0^+}{\tilde{r}^+} e$$

**Compression:**

$$d^- = \tilde{r}^- \frac{\partial G^-(\tilde{r}^-)}{\partial \tilde{r}^-} = \dot{G}^- \geq 0$$

$$d^- = 1 - \frac{r_0^-}{\tilde{r}^-} (1 - A^-) - A^- e$$

Olivier, J.; Cervera, M.; Oller, S.; Lubliner, J. (1990), Isotropic Damage Models and Smeared Crack Analysis of Concrete, Proc. 2nd Int. Conf. on Comp. Aided Analysis and Design of Conc. Struct., Zell am See, Austria

Evolution of plastic strain tensor

$$\dot{\varepsilon}^p = \beta EH (\dot{d}^-) \frac{\langle \sigma : \dot{\varepsilon} \rangle}{\sigma : \sigma} D_0^{-1} : \sigma$$

$$\beta - \text{material parameter} : \quad \varepsilon^p = \beta (\varepsilon - \varepsilon_0)$$

$$H(\dot{d}^-) - \text{Heaviside function}$$

**Cauchy stress**

$$\sigma = \frac{\partial \Psi}{\partial \varepsilon}; \quad \varepsilon = \varepsilon^e + \varepsilon^p$$

$$\frac{\partial \Psi}{\partial \varepsilon^e} = \sigma = (1 - d^+) \sigma^+ + (1 - d^-) \sigma^-$$

Yankelevsky, D.Z.; Reinhardt, H.W (1987), Model for Cyclic Compressive Behavior of Concrete, J. of Struct. Eng., ASCE, Vol. 113
Examples of numerical analysis

1) plate without fibres, impact of 40 kg (height 0.25 m).

Upper view

Examples of numerical analysis

1) plate without fibres, impact of 40 kg (height 0.25 m).

Bottom view.
Examples of numerical analysis

2) plate with fibres, impact of 200 kg (height 0.25 m).
Upper view.

Examples of numerical analysis

2) slab with fibres, impact of 200 kg (height 0.25 m).
Bottom view.
CONCLUSIONS

This study shows the necessity of experimental verification for numerical analysis performed with the use of advanced nonlinear finite element algorithms. Among many important factors for such analysis the assumption of adequate material model for concrete describing the entire dynamic response: from initial pure elastic behaviour until the total material damage is the most important factor for the adequacy of numerical results.

For lower values of fibre percentage in the material, the localization of large cracks has been observed. In this case the application of XFEM (eXtended Finite Element Method) is now under investigation in Abaqus/Standard environment.

The expected results of this investigation are not only limited to prepared recipes for the concrete, but also concern the elaborated and calibrated material models, applicable in nonlinear finite element computer codes.

References